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From D-$\overline{\text{D}}$ pairs to branes in motion

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Abstract: We investigate various supersymmetric brane intersections. Motivated by the recent results on supertubes, we investigate general constraints in which parallel brane-antibrane configurations are supersymmetric. Dual descriptions of these configurations involve systems of intersecting branes in relative motion. In particular, we find new supersymmetric configurations which are *not* related to a static brane intersection by a boost. In these new configurations, the intersection point moves at the speed of light. These systems provide interesting time-dependent backgrounds for open strings.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

Contents

1. Introduction

D-branes [1] have played a central role in advancing our understanding of string theory in the past seven years, and despite the impressive collection of results that has accumulated about these objects — see, e.g., ref. [2] — they continue to reveal new surprises. For example, Townsend and Mateos [3] discovered a supersymmetric configuration in which a cylindrical D2-brane is supported against collapse by the angular momentum of crossed electric and magnetic fields on its worldvolume. As a result of the worldvolume gauge fields, these supertubes also lend themselves to an interpretation as bound states of a cylindrical D2-brane with D0-branes and F-strings. In particular, the latter constituents seem to define the supersymmetries preserved by the supertube. An essential ingredient in this result seems to be that the electric field takes the 'critical' value

$$E = \pm \frac{1}{2\pi l_s^2}\,. \tag{1.1}$$

While not producing a vanishing brane tension, this critical field further allows the cross-section of the cylinder to take an arbitrary profile [4]. In any event, as the cross-section is closed, the system carries no overall D2 charge. Roughly antipodal elements of the cylinder have the opposite orientation and so can be considered as D2-$\overline{\text{D2}}$ pairs for which the charge cancels. From this point of view, the fact that the configuration is supersymmetric comes as a surprise.

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Ref. [5] demonstrated, in the matrix theory description of the supertube [6], that a one-quarter supersymmetric, parallel D2-$\overline{\text{D2}}$ pair could be obtained as the limit of a supertube of elliptical cross-section, in which the major axis is taken to be infinitely large while the minor axis is held fixed. In order to preserve charge, the tube necessarily degenerates into a brane-antibrane pair. Again, such a configuration typically would break all background supersymmetry, however, as for the supertube, partial supersymmetry is retained because of the presence of worldvolume gauge field fluxes. Ref. [5] discusses the supersymmetry and stability of these configurations in the Born-Infeld, supergravity and matrix theory descriptions, while in ref. [7], similar issues are analysed from the worldsheet perspective of open strings connecting the branes. It is shown that the tachyonic mode which might naively appear at small brane separations is lifted from the spectrum when the supersymmetry conditions are satisfied. A similar discussion appears in ref. [4], while ref. [8] provides a similar analysis for D3-$\overline{\text{D3}}$ pairs. The electric fields considered in refs. [4, 5, 7, 8] are chosen to take the critical value (1.1), which is natural, given that their motivation comes from supertube physics.

In section 2, we examine the general supersymmetry conditions for parallel D2-D2 and D2-$\overline{\text{D2}}$ pairs with gauge field fluxes. More precisely, we consider parallel brane configurations in flat space and find a three parameter family of worldvolume gauge fields that preserve a quarter of the thirty-two background supersymmetries. In particular, we show that a 'critical' electric field (1.1) is not a necessary condition. A similar result is implicit in the analysis of D3-$\overline{\text{D3}}$ and D4-$\overline{\text{D4}}$ pairs found in ref. [9].

In section 2.2, we investigate several dual descriptions of the above configurations. The lift to eleven-dimensional M-theory produces a pair of M2-branes oriented at an angle to one another and also in relative motion. Similarly, T-duality along the direction of the electric fields produces angled D-strings in relative motion. In both of these cases, the surprise is that supersymmetry is preserved even though the branes are moving relative to one another.

Hence, in section 3, we provide a more general discussion of branes in motion. An important ingredient in producing a supersymmetric configuration is that the intersection point travels at the speed of light. We note that such null intersections were considered in ref. [10] in an M-theory framework. We investigate these configurations from different physical points of view, and find that although they are supersymmetric, their perturbations seem unstable. We find that the open string excitations connecting the branes cannot typically follow the intersection point and so are stretched as they fall behind.

The main text closes with a brief discussion of our results in section 4. Appendix A presents some technical details involved in the T-duality transformations appearing in section 2.2.

2. D-$\overline{\text{D}}$ pairs

The primary focus of this section is to examine general constraints that arise from demanding that a parallel pair consisting of a D-brane and an anti-D-brane is supersymmetric. While naively such a configuration breaks all of the supersymmetries, the system can be restored to being one-quarter BPS by introducing fluxes of the worldvolume gauge fields [4, 5]. As in ref. [9], and in contrast to most previous analyses, our generalized approach shows that the supersymmetric configurations need not involve 'critical' electric fields (1.1). The latter result could have been anticipated by noting that the condition in eq. (1.1) is not invariant under Lorentz boosts in the worldvolume.

The following calculations focus on the supersymmetry conditions for parallel D2-D2 and D2-$\overline{\text{D2}}$ pairs with gauge field fluxes. We also restrict ourselves to cases in which the electric fields on each brane can be Lorentz-transformed to lie in the same direction.

2.1 SUSY pairs

As described in ref. [11], the condition for a D-brane to preserve some of the supersymmetry of its background is based upon the invariance of the worldvolume action under local κ-symmetry and global spacetime supersymmetry transformations of the spacetime fermion fields. Making a combination of these transformations, and gauge fixing the κ-symmetry, results in a global worldvolume supersymmetry. By requiring consistency of the gauge choice under further supersymmetry transformations, one finds a constraint on the Killing spinors, ϵ, that generate the supersymmetries of the background. This demands that the generators satisfy $\Gamma\epsilon = \pm\epsilon$, where Γ is a hermitean traceless product structure (i.e., Tr $\Gamma = 0$ and $\Gamma^2 = 1$) appearing in the κ-symmetry transformation. The sign on the right-hand side of this condition is taken to be positive for a brane and negative for an antibrane. For a D2- or $\overline{\text{D2}}$-brane, the product structure is given by

$$\Gamma = \frac{1}{\sqrt{-\det(g+\mathcal{F})}}\left(1 + \frac{1}{2}\hat{\Gamma}^{ab}\mathcal{F}_{ab}\Gamma_{11}\right)\hat{\Gamma}_{012}\,, \tag{2.1}$$

where $\mathcal{F} = B + 2\pi\ell_s^2 F$ is the generalised Born-Infeld (BI) field strength and $\hat{\Gamma}_a$ denote the 'pull-backs' of the spacetime Dirac matrices to the worldvolume — see eq. (2.3) below. If we choose a background coordinate system, x^μ, then the induced metric on the worldvolume can be written in terms of the ten-dimensional *zehnbein* $E^{\underline{\alpha}}_{\ \mu}$, as

$$g_{ab} = \partial_a x^\mu \partial_b x^\nu G_{\mu\nu} = (\partial_a x^\mu E^{\underline{\alpha}}_{\ \mu})(\partial_b x^\nu E^{\underline{\beta}}_{\ \nu})\eta_{\underline{\alpha\beta}}\,. \tag{2.2}$$

Then, the $\hat{\Gamma}_a$ are defined as

$$\hat{\Gamma}_a = (\partial_a x^\mu E^{\underline{\alpha}}_{\ \mu})\Gamma_{\underline{\alpha}}\,, \tag{2.3}$$

where $\Gamma_{\underline{\alpha}}$ are the constant, ten-dimensional Dirac matrices with $\{\Gamma_{\underline{\alpha}}, \Gamma_{\underline{\beta}}\} = 2\eta_{\underline{\alpha\beta}}$. By $\Gamma_{11} = \Gamma_0, \ldots, \Gamma_9$ we denote the ten-dimensional chirality operator. In the analysis that follows we begin by considering a pair of parallel D2-branes carrying fluxes. The extension to a D2-$\overline{\text{D2}}$ pair appears at the end of the section — all pertinent results are presented, but the details omitted since the analyses are so similar.

We work exclusively in a Minkowski space background. We adopt a cartesian coordinate frame, so the background Killing spinors ϵ are constant and arbitrary (up to being non-zero). Also, the *zehnbein* is trivially $E^{\underline{\alpha}}_{\ \mu} = \delta^{\underline{\alpha}}_{\ \mu}$ so that $\hat{\Gamma}_a = (\partial_a x^\mu)\Gamma_\mu$. Hence, we see that for a flat D2-brane aligned, in static gauge, with the t, x and y directions of this background, we have $\hat{\Gamma}_0 = \hat{\Gamma}_t = \Gamma_t$, $\hat{\Gamma}_1 = \hat{\Gamma}_x = \Gamma_x$, $\hat{\Gamma}_2 = \hat{\Gamma}_y = \Gamma_y$ and that eq. (2.1) reduces to

$$\Gamma = \frac{1}{\sqrt{-\det(\eta + \mathcal{F})}} \left(\Gamma_{txy} + (\Gamma_y \mathcal{F}_{tx} + \Gamma_x \mathcal{F}_{yt} + \Gamma_t \mathcal{F}_{xy})\Gamma_{11}\right) . \tag{2.4}$$

Due to the properties of Γ, the single condition $\Gamma\epsilon = \epsilon$ reduces the number of independent components of ϵ by a factor of two. That is, a single D2-brane with arbitrary worldvolume fluxes is one-half BPS. For a multi-brane configuration, one tests the mutual compatibility of the single-brane conditions. In general, this reduces the number of independent components of the surviving Killing spinor ϵ to k, say, in which case the fraction of supersymmetry preserved is $k/32$. Incompatibility of the single-brane conditions implies $k = 0$, meaning that supersymmetry is completely broken.

We consider a pair of flat, parallel D2-branes parametrised with the background coordinates (t, x, y), as above. They may be separated by a finite distance in the transverse dimensions. In addition, they have constant BI field strengths of the form

$$F_i = E_i \, \mathrm{d}y \wedge \mathrm{d}t + B_i \, \mathrm{d}y \wedge \mathrm{d}x \, , \qquad (i = 1, 2) \tag{2.5}$$

while we assume that the Kalb-Ramond field vanishes. This ansatz incorporates many possible field configurations. In particular, any configuration in which at least one of the branes, say the second, has $F_2 \cdot F_2 \neq 0$ is Lorentz-equivalent to one of the form (2.5) with either $E_2 = 0$ (if $F_2 \cdot F_2 > 0$) or $B_2 = 0$ (if $F_2 \cdot F_2 < 0$). In many cases it is also possible to boost to a frame in which either E_1 or E_2 takes the critical value, but we will not restrict ourselves in this way.

Given this ansatz for the fields, the supersymmetry conditions become, from eq. (2.4),

$$\Gamma_i \epsilon = \frac{1}{\mathcal{L}_i} (\Gamma_{txy} + 2\pi\ell_s^2 \, E_i \Gamma_x \Gamma_{11} - 2\pi\ell_s^2 \, B_i \Gamma_t \Gamma_{11})\epsilon = \epsilon \, , \qquad (i = 1, 2) \tag{2.6}$$

where $\mathcal{L}_i = \sqrt{-\det(\eta + F_i)} = \sqrt{1 - 4\pi^2\ell_s^4 \, E_i^2 + 4\pi^2\ell_s^4 \, B_i^2}$. The commutator of Γ_1 and Γ_2 provides the useful result

$$[\Gamma_1, \Gamma_2]\epsilon = 0 \;\Leftrightarrow\; \left((E_1 - E_2)\Gamma_{ty}\Gamma_{11} - (B_1 - B_2)\Gamma_{xy}\Gamma_{11} + 2\pi\ell_s^2 (E_1 B_2 - E_2 B_1)\Gamma_{xt}\right) \epsilon = 0 \, . \tag{2.7}$$

Using eqs. (2.6) and (2.7), and the properties of the gamma operators, we find that

$$\Gamma_y \epsilon = \frac{4\pi^2\ell_s^4 (E_1 B_2 - E_2 B_1)}{\mathcal{L}_2 - \mathcal{L}_1}\epsilon = \frac{B_2 \mathcal{L}_1 - B_1 \mathcal{L}_2}{E_1 - E_2}\epsilon = \frac{E_2 \mathcal{L}_1 - E_1 \mathcal{L}_2}{B_1 - B_2}\epsilon \, . \tag{2.8}$$

Further, we note that the eigenvalues of Γ_y are ± 1, as $\Gamma_y^2 = 1$. Therefore the above seems to provide three constraints on the four independent field strengths. Remarkably, however, one finds that there remains a three parameter family of solutions, as follows. For each eigenvalue of Γ_y, eq. (2.8) provides three expressions for, say, E_1. Each of these is a solution to a quadratic equation and so contains an arbitrary sign. For a suitable choice of these signs, the three expressions are identical. This gives the most general solution for E_1, as a function of the other three fields. Hence, we have the auxiliary projection

$$\Gamma_y \epsilon = \pm \epsilon \, , \tag{2.9}$$

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which applies with the matching field configuration

$$E_1 = E_1^{\pm} \equiv \frac{E_2(1+4\pi^2\ell_s^4 B_1 B_2) \pm (B_2 - B_1)\mathcal{L}_2}{1+4\pi^2\ell_s^4 B_2^2}\,. \tag{2.10}$$

Here one simultaneously chooses either the plus or the minus sign in each of these equations. These two expressions follow from eqs. (2.6) and so are necessary for supersymmetry, but they may not be sufficient to ensure it. To check this we must find under what additional conditions, if any, the imposition of eqs. (2.9) and (2.10) guarantees the compatibility of eqs. (2.6). By considering each supersymmetry condition in turn we find that they are actually equivalent, and not merely compatible, if

$$\Gamma_y \epsilon = \pm\epsilon\,, \quad E_1 = E_1^{\pm}\,, \quad \text{and} \quad (1+4\pi^2\ell_s^4\, B_1 B_2)\mathcal{L}_2 \pm 4\pi^2\ell_s^4\, E_2(B_1 - B_2) > 0\,. \tag{2.11}$$

Hence, the amount of supersymmetry preserved by these branes is determined by the compatibility of the operators Γ_y and Γ_1, say, given that we choose a valid field configuration from the region of parameter space allowed by the inequality. We note that, like Γ_1, Γ_y is traceless and squares to the identity. Furthermore, it can easily be checked that the two operators commute. By solving the two conditions simultaneously we find that 1/4 of the background supersymmetry is preserved. What is more, the simple form of the auxiliary projection is virtually independent of the brane configuration (the only dependence being that it projects into the volume of the brane). This means that the same conditions can be applied to find supersymmetric configurations of any number of parallel branes, so long as each of the field strengths takes the form (2.5) and is related to that on an arbitrarily chosen reference brane by eq. (2.10).

This entire calculation can be easily modified to consider the case of a parallel D2-$\overline{\text{D2}}$ pair. In the heuristic picture of an anti-D2 brane as an 'upside-down' D2-brane, we see that the transition from a D2 to an anti-D2 involves a parity transformation of one of the worldvolume coordinates, in which case $\Gamma \to -\Gamma$. As advertised above, the supersymmetry condition for a flat anti-D2 brane is therefore $\Gamma\epsilon = -\epsilon$, with Γ as in eq. (2.4). Assuming an ansatz of the form (2.5), with F_2 on the antibrane, the supersymmetry conditions become

$$\Gamma_i \epsilon = \frac{1}{\mathcal{L}_i}(\Gamma_{txy} + 2\pi\ell_s^2\, E_i \Gamma_x \Gamma_{11} - 2\pi\ell_s^2\, B_i \Gamma_t \Gamma_{11})\epsilon = s_i \epsilon\,, \qquad (i = 1,2) \tag{2.12}$$

where $s_1 = 1$ and $s_2 = -1$. A repetition of the above calculation shows that one-quarter supersymmetric, parallel D2-$\overline{\text{D2}}$ pairs exist under the conditions

$$\begin{aligned} \Gamma_y \epsilon = \pm\epsilon\,, \qquad & E_1 = \widetilde{E}_1^{\pm} \equiv \frac{E_2(1+4\pi^2\ell_s^4\, B_1 B_2) \mp (B_2 - B_1)\mathcal{L}_2}{1+4\pi^2\ell_s^4\, B_2^2} \\ \text{and} \qquad & (1+4\pi^2\ell_s^4\, B_1 B_2)\mathcal{L}_2 \mp 4\pi^2\ell_s^4\, E_2(B_1 - B_2) < 0\,. \end{aligned} \tag{2.13}$$

Note the subtle difference in signs between these expressions and those appearing in eq. (2.11). Just as in the D2-D2 case, these results imply the equivalence of the single-brane conditions and so can be used to establish the supersymmetry of a collection of arbitrarily many branes and antibranes by relating the gauge fields on each (anti)brane to those on a reference brane using the appropriate expression, eq. (2.10) or eq. (2.13). A similar configuration of branes and antibranes was studied in ref. [5], in which all the electric fields are critical. In that case the system explicitly preserves the supersymmetries corresponding to the presence of F-strings and D0-branes. That is, the Killing spinors satisfy

$$\text{F1:} \quad \Gamma_{ty}\epsilon = -\Gamma_{11}\epsilon \qquad \text{and} \qquad \text{D0:} \quad \Gamma_t\Gamma_{11}\epsilon = \epsilon\,. \tag{2.14}$$

In our conventions, the supersymmetric field choices of ref. [5] are

$$2\pi\ell_s^2 E_1 = -1\,, \qquad B_1 < 0\,, \qquad 2\pi\ell_s^2 E_2 = -1 \qquad \text{and} \qquad B_2 > 0\,. \tag{2.15}$$

Substituting the latter three into the conditions (2.13) as parameters, one finds the $\Gamma_y\epsilon = \epsilon$ solution $2\pi\ell_s^2 E_1 = -1$, so these choices are consistent with our analysis. The auxiliary projection emerges trivially from manipulating the F-string and D0-brane supersymmetry projections (2.14).

2.2 Dual descriptions

We now describe various dual interpretations of the brane configurations discussed above. These are gained by the usual methods of T-duality, and by lifting to M-theory.

Consider the transition from ten to eleven dimensions. The gauge degrees of freedom on a D-brane are traded for geometrical degrees of freedom describing the orientation and velocity of the dual M2-brane in the eleventh dimension. The details of this interchange, made explicit in the formal derivation of the M2-brane action from the D2-brane action [12, 13], are (assuming a flat space background)

$$\partial_a z = \frac{\pi\ell_s^2\,\epsilon_{abc}F^{bc}}{\sqrt{1+2\pi^2\ell_s^4\,F^2}}\,, \tag{2.16}$$

where z is the eleventh dimension and F is the BI field strength.[1] The partial derivatives are with respect to the worldvolume coordinates on the D2-brane.

To illustrate this, we consider a supersymmetric pair of D2-branes with worldvolume gauge fields

$$E_1 = B_1 = 0\,, \qquad E_2 = \pm B_2 = -\varepsilon\,. \tag{2.17}$$

One may verify that this choice is compatible with eq. (2.11). According to eq. (2.16), the D2-brane carrying the fluxes lifts to an M2-brane with

$$\partial_x z = \pm\partial_t z = 2\pi\ell_s^2\varepsilon\,, \tag{2.18}$$

while the other simply becomes a flat, stationary M2-brane with $z = 0$. Hence, one of the M2-branes is rotated and boosted into the eleventh dimension, while the other remains at rest in the (x, y) plane. Note that the branes may not intersect, as they may be displaced in the overall transverse dimensions. Supersymmetry is preserved in the M-theory lift, so these M2-branes remain 1/4-supersymmetric. This is an interesting result, as it is well

[1] In general, the pull-back of the RR one-form also enters this expression but we have set it to zero.

known that, generically, branes at angles do not preserve supersymmetry. Exceptional classes of supersymmetric branes exist [14, 15, 16], however, they would involve rotating the branes simultaneously in at least two orthogonal planes. (This need not be true for bound states, as we will see below.) In contrast, the second M2-brane above is only rotated in the (x, z) plane relative to the first and so the additional motion of this brane must be essential to preserving supersymmetry. We note that the speed with which the intersection point between the two branes travels (or rather the speed of the intersection of the moving M2-brane with a constant z hypersurface) is easily determined from eq. (2.18) to be

$$\left.\frac{dx}{dt}\right|_I = \frac{\partial_t z}{\partial_x z} = \pm 1 \,. \tag{2.19}$$

As we will discuss in the next section, the fact that the brane intersection moves at the speed of light is a necessary condition for supersymmetry. Similar observations were made in classifying the most general supersymmetric configurations of two M5-branes [10].

Note that if we choose $B_1 = B_2$, the only solution for the electric fields is $E_1 = E_2$. In lifting these to M-theory they are therefore rotated by the same angle and become parallel M2-branes. This indicates more generally that non-parallel M2-branes can only preserve supersymmetry if they are in relative motion. A final comment is that, clearly, by returning to ten dimensions by compactifying a direction transverse to all of the worldvolume directions, one recovers identical configurations of D2-branes with vanishing gauge fields.

Other related brane configurations can be found using T-duality. Of course, dualising in the transverse directions will simply produce higher-dimensional versions of those described above. More interesting are the T-dualities along the directions parallel to the branes, which we discuss here.

A D2-brane with a general BI field strength is interpreted as a D2-D0-F1 bound state and T-dualising parallel to its worldvolume produces a D1-F1 bound state with momentum in the plane of the initial system (see the appendix for details). This observation allows one to relate the supertube configurations to a super-helix [17, 18] via a T-duality along the axis of the cylinder. A supersymmetric pair of parallel D2-branes, as discussed above, will therefore have a (supersymmetric) dual that consists of two D-strings in relative motion. In general the D-strings will *not* be parallel, so here we again see the preservation of supersymmetry in an unusual setting, involved with the motion of the branes. A more detailed discussion of these configurations will be postponed to section 3.

For our D2-D2 solutions in section 2.1, momentum arises from T-duality along the y axis, and the D-strings carry no electric flux, i.e., they are pure D-strings, rather than D1-F1 bound states. If we T-dualise along the x axis, on the other hand, we find static configurations of angled D1-F1 bound states, or $(p, 1)$-strings. This is in contrast to the situation in M-theory, where relative motion is an essential ingredient in the supersymmetry of non-parallel M-branes. As an example, consider again the simple gauge field configuration (2.17). The brane with vanishing gauge fields T-dualises to a stationary $(0, 1)$-string (i.e., a D-string) parallel to the y axis. The other T-dualises to a $(p, 1)$-string that is tilted in the (x, y) plane. The tilt angle, θ, with respect to the x axis, and the electric field, e, on the D-string, are related to E_1 and B_1 by

$$E_1 = \frac{e}{\sin\theta}\,, \qquad B_1 = \frac{\cot\theta}{2\pi\ell_s^2}\,. \tag{2.20}$$

Therefore, the system of D-strings is supersymmetric when

$$e = \pm\frac{\sin\phi}{2\pi\ell_s^2}\,, \tag{2.21}$$

expressed in terms of $\phi = \pi/2 - \theta$, which is the angle between the two branes. This is in agreement with the standard result, $e = 0$, for parallel D-strings, and implies that orthogonal D-strings are supersymmetric for $|e| = (2\pi\ell_s^2)^{-1}$, i.e., critical electric flux. Since e is the only component of the BI field strength on the D-string, this would signal the vanishing of its tension.

It is interesting to consider the case of intersecting D-strings, i.e., the configurations where the D-strings are not displaced in the overall transverse directions. In this case, we can relate the results to discussions on string networks [19, 20, 21]. In particular, the intersecting D-string configuration above can be thought of as a special case of a two vertex string network, in which the string joining the two vertices has been shrunk to zero length — see figure 1. When each string of type (p, q) has the same orientation within the network as all the other strings of that type, the network is 1/4-supersymmetric [19, 20, 21]. From the figure, it can be seen that the two vertex network indeed satisfies this condition. We have chosen the orientations and charges such that, when the internal string is shrunk to zero length, the diagram reduces to the D-string configuration discussed above.

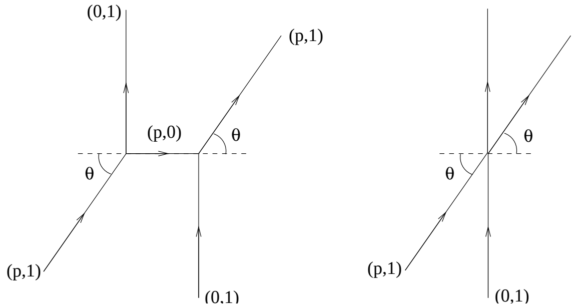

Figure 1: The two vertex string network and the limiting case in which the internal $(p, 0)$-string has been shrunk to zero length.

To verify this interpretation, we shrink the connecting $(p, 0)$-string in figure 1 to recover the crossed D-strings without changing θ. The latter should then be related to the flux on the $(p, 1)$-string by the supersymmetry condition (2.21): $2\pi\ell_s^2|e| = |\cos\theta|$. Standard results [20] on three string junctions allow the angle θ to be expressed as

$$\cos\theta = \frac{|p|g_s}{\sqrt{p^2 g_s^2 + 1}}\,, \tag{2.22}$$

so our interpretation is valid if the electric flux on the $(p, 1)$-string is related to p by

$$2\pi\ell_s^2|e| = \frac{|p|g_s}{\sqrt{p^2 g_s^2 + 1}}\,. \tag{2.23}$$

The latter is verified by a standard calculation relating the number of F-strings to the electric displacement on the D-string — see, e.g., ref. [22]. Moreover, the two configurations preserve the same amount of supersymmetry, so we conclude that our interpretation is correct.

It was argued earlier that our results could also be used to find supersymmetric stacks of more than two parallel D2-branes. The above discussion of dual configurations can therefore be extended to include an arbitrary number of intersecting D-strings. Of course, T-dualities perpendicular to the plane of the D-strings will produce supersymmetric systems of intersecting Dp-branes. The Dp-branes will inherit an identical electric field from the corresponding D-string. Hence the supersymmetry condition will be the same as eq. (2.21).

3. Branes in motion

The presence of non-trivial gauge fields preserving supersymmetry has been seen to produce interesting systems dual to the D2-D2 or D2-$\overline{\text{D2}}$ pairs, both in string theory and M-theory. In both cases the relevant duality generally mixes, or replaces, the fluxes with angles and *momenta*. This motivates a discussion in this section of more general configurations where the constituent branes are in relative motion. It is useful to consider the action of Lorentz boosts on D-branes, so we provide the following discussion.

3.1 Dirichlet meets Lorentz (and Pythagorus)

The essential observation is that a(n unexcited) D-brane cannot support any momentum in directions along its worldvolume. Hence consider an observer moving at an angle to a D-brane, as illustrated in figure 2a. If one boosts to the frame of reference of this observer (see figure 2b), one finds that the spatial momentum vector describing the moving D-brane is orthogonal to the angled brane. That is, the D-brane momentum is *not* antiparallel to the observer's original velocity.

Of course, this amusing result arises because of the relativistic nature of the D-brane's stress energy tensor. We can make precise the qualitative observations above by considering the application of a boost to the stress energy tensor. We will frame the following discussion in the context of a D-string angled in the (x, y) plane, as shown in the figure. The same formulae obviously apply for a higher-dimensional D-brane extending off in orthogonal directions, since only the components in the (t, x, y) directions will be relevant. Furthermore, we miss nothing by omitting the δ-functions which may localise the brane's stress-energy tensor in some of these additional directions.

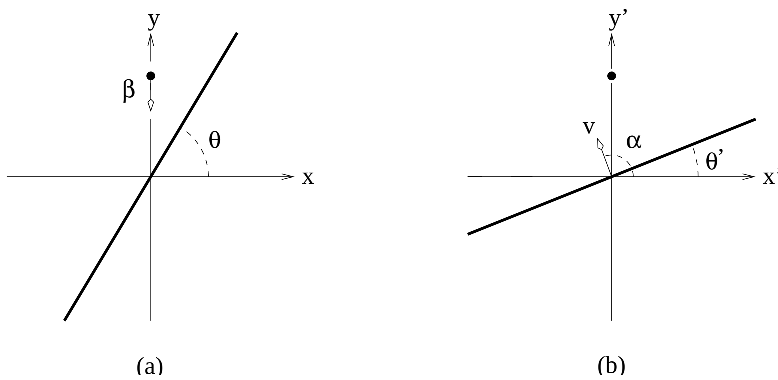

Figure 2: Two perspectives on an angled brane.

The stress energy tensor for a D-brane at rest at an angle θ to the x axis is easily derived to be

$$T_{ab} = \mathrm{T} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos^2\theta & -\cos\theta\sin\theta \\ 0 & -\cos\theta\sin\theta & -\sin^2\theta \end{bmatrix} \delta(y\cos\theta - x\sin\theta)\,, \tag{3.1}$$

where T is the brane tension. It is easy to see that for $\theta = 0$ this expression reduces to a diagonal matrix, $T_{ab} = \mathrm{T}\, diag(1,-1,0)\,\delta(y)$, as expected for a D-brane lying along the x axis. Now applying a boost along the y axis with velocity β, so as to bring the observer in figure 2a to rest, one finds

$$T_{a'b'} = \mathrm{T} \begin{bmatrix} \gamma^2(1-\beta^2\sin^2\theta) & \beta\gamma\sin\theta\cos\theta & -\beta\gamma^2\cos^2\theta \\ \beta\gamma\sin\theta\cos\theta & -\cos^2\theta & -\gamma\sin\theta\cos\theta \\ -\beta\gamma^2\cos^2\theta & -\gamma\sin\theta\cos\theta & \gamma^2(\beta^2-\sin^2\theta) \end{bmatrix} \delta(\gamma(y'-\beta t')\cos\theta - x'\sin\theta)\,, \tag{3.2}$$

where as usual $\gamma = 1/\sqrt{1-\beta^2}$. Note that for $\theta = \pi/2$, the stress energy reduces to $T_{a'b'} = \mathrm{T}\, diag(1,0,-1)\,\delta(x')$. That is, this corresponds to a D-brane lying on the y axis and, since the boost is then in the worldvolume of the (unexcited) brane, the configuration and the stress energy tensor are left invariant. In general, from the off-diagonal components $T_{t'i'}$ one sees that the boosted brane has a momentum density with components along both the x' and y' axes, even though the boost was along the y' axis. The angle of the motion with the x' axis is given by

$$\tan\alpha = \frac{T_{t'y'}}{T_{t'x'}} = -\frac{\gamma}{\tan\theta}\,, \tag{3.3}$$

while the angle which the brane makes with the x' axis may be read off from the argument of the δ-function to be

$$\tan\theta' = \frac{\tan\theta}{\gamma}\,, \tag{3.4}$$

at any instant (in time t'). Hence we have $\tan\alpha\,\tan\theta' = -1$, indicating that, as expected, the spatial momentum of the boosted brane is orthogonal to its spatial worldvolume. One observation about this geometry is that even though the brane moves off at an angle α in the (x',y') plane, the intersection point of the brane with the y' axis ($x' = 0$) moves with precisely the expected velocity β. One final comment is that both the static and boosted branes will be equally supersymmetric.

As mentioned above, this 'spontaneous' generation of momentum orthogonal to the boost direction applies for any extended relativistic brane boosted at an angle to its worldvolume. This includes D-branes, M-branes, fundamental strings and NS5-branes. The effect manifests itself in interesting ways in configurations dual to an angled brane. We briefly consider two examples here.

Consider 'T-dualising' a D-string, angled as in figure 2a, along the x axis.[2] As already implicit in the discussion of section 2.2, the dual configuration corresponds to a D2-brane filling the (x,y) plane and carrying a uniform magnetic flux — see, e.g., refs. [2, 23]. The

[2]Note that, in the following, we implicitly do not consider the x coordinate to be compact for either configuration and so they are not, strictly speaking, T-dual to each other.

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latter flux may be interpreted as constant density of D0-branes dissolved in the worldvolume of the D2-brane:

$$\rho_{\rm D0} = \frac{1}{2\pi} F_{yx} = \frac{\cot\theta}{(2\pi\ell_s)^2} \,. \tag{3.5}$$

In this case, because of the worldvolume excitation, i.e., the magnetic field, the system is not invariant under boosts parallel to the D2-brane. Rather, a boost in, say, y, generates an electric field

$$F_{x't'} = \gamma\beta\frac{\cot\theta}{2\pi\ell_s^2}\,, \qquad F_{y'x'} = \gamma\frac{\cot\theta}{2\pi\ell_s^2}\,. \tag{3.6}$$

The appearance of the electric field corresponds to the 'spontaneous' generation of a density of fundamental strings aligned along the x' axis:

$$\rho_{\rm F1} = \frac{\gamma\beta\cos\theta}{2\pi\ell_s g_s}\,, \tag{3.7}$$

corresponding to the electric displacement calculated from the Born-Infeld action — see, e.g., ref. [22]. Note that in this boosted configuration as well as winding number along the x' axis, there is momentum along the y' axis since $T_{t'y'} \propto F_{t'x'}F_{y'x'}$ is nonvanishing. The presence of both of these is expected, in order to match the T-dual configuration, i.e., the boosted D-string carrying both x' and y' components of momentum density. This is only possible because the operations of Lorentz boost and T-duality commute if they are performed at right-angles to one another. Of course, this subtle interplay fails when both the T-duality and the boost are made in the same direction. Then the two operations do not commute, as the momentum generated by a boost in a certain direction is affected by the subsequent action of T-duality in that direction. These assertions may be verified explicitly by the reader, using the T-duality relations given in the appendix.

Another interesting case considered in section 2.2 is to reinterpret a D2-brane with fluxes in terms of its lift to M-theory, as described by eq. (2.16). The D0-D2 bound state above, with only a magnetic flux, corresponds to a M2-brane moving in the eleventh dimension. Boosting along the y axis introduces an electric field parallel to the x axis on the D2-brane, which corresponds to tilting the M2-brane in the (z,y) plane. (Recall that z is the eleventh dimension.) Note that, just as the magnetic field on the D2-brane is a worldvolume excitation and so causes the D-brane to 'feel' the boost, the motion of the original M2-brane orthogonal to the boost axis can also be regarded as a worldvolume excitation and, as a result, the system is not invariant even though the boost is 'along the brane' as seen at any given instant of time.

Alternatively, one could consider beginning with only an electric flux F_{xt}, i.e., a D2-F1 bound state, in which case a boost in y introduces a magnetic flux F_{yx}. The M-theory description is then analogous to the description of boosting a tilted D-brane given above. One begins with an M2-brane tilted in the (z,y) plane and boosting along the y axis spontaneously generates a component of momentum in the z direction.

The next observation becomes useful in describing multiple brane configurations involving relative motion of the branes. Consider two branes oriented in the (x,y) plane as in figure 3*a*. The first is static and lies along the x axis. The second is oriented at an angle

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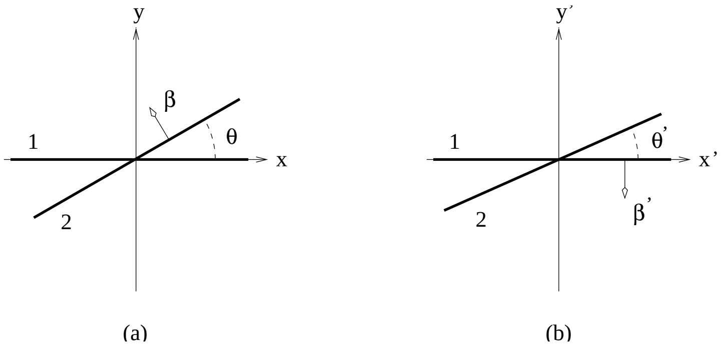

Figure 3: The moving brane intersection from (a) the rest frame of brane 1, (b) the rest frame of brane 2.

θ with the x axis and moves with a velocity β (which is directed orthogonal to the brane). Now the apparent velocity of the intersection point[3] along the x axis is

$$\beta_I = -\frac{\beta}{\sin\theta}\,, \tag{3.8}$$

which comes from a simple geometric projection of the brane's motion on the x axis. The interesting point is that while β is a physical velocity and so has magnitude less than 1, there is no such bound on this apparent velocity. In fact, β_I reaches the speed of light for $\sin\theta = \beta$ and becomes superluminal for $\sin\theta < \beta$. Note in particular that $|\beta_I| > 1$ may involve an arbitrarily small physical velocity β as long as the intersection angle is correspondingly small.

As may be inferred from the comments at the beginning of the section, one can easily boost the configuration with $|\beta_I| < 1$ to a static configuration of intersecting branes by boosting along the x axis by $-\beta_I$, i.e., one makes a boost along the static brane. It is amusing then to note that from the point of view of an observer on the second brane one reaches the static configuration by boosting along the worldvolume of her brane — see figure 3b. Of course, there is no inconsistency here. In any frame of reference, one can simply follow the trajectory of the intersection point

$$\vec{x}(t) = \vec{x}_0 + \vec{\beta}_I t\,, \tag{3.9}$$

and then a boosting by $-\vec{\beta}_I$ will bring the intersection to rest, i.e., $\vec{x}'(t) = \vec{x}'_0$. Naturally, this procedure only works when $|\vec{\beta}_I| < 1$ — see figure 4a.

In the case $|\vec{\beta}_I| > 1$, the trajectory (3.9) lies outside the lightcone and so one cannot boost to a static configuration. However, it is possible to boost so that the intersection becomes a line on a constant time slice. The necessary operation would be boosting by $-\vec{\beta}_I/|\vec{\beta}_I|^2$, e.g., in the above example, one would boost along the x axis by a velocity $-1/\beta_I = \sin\theta/\beta$. In terms of the full brane configuration, this boost brings one to a frame where the two D-strings are parallel, with one moving with positive velocity along the y axis. The intersection then occurs at the instant where this brane crosses the x' axis. This is shown in figure 4b.

[3]Of course, the branes need not intersect if they are displaced in the overall transverse directions, however, we adopt this nomenclature even in describing such cases.

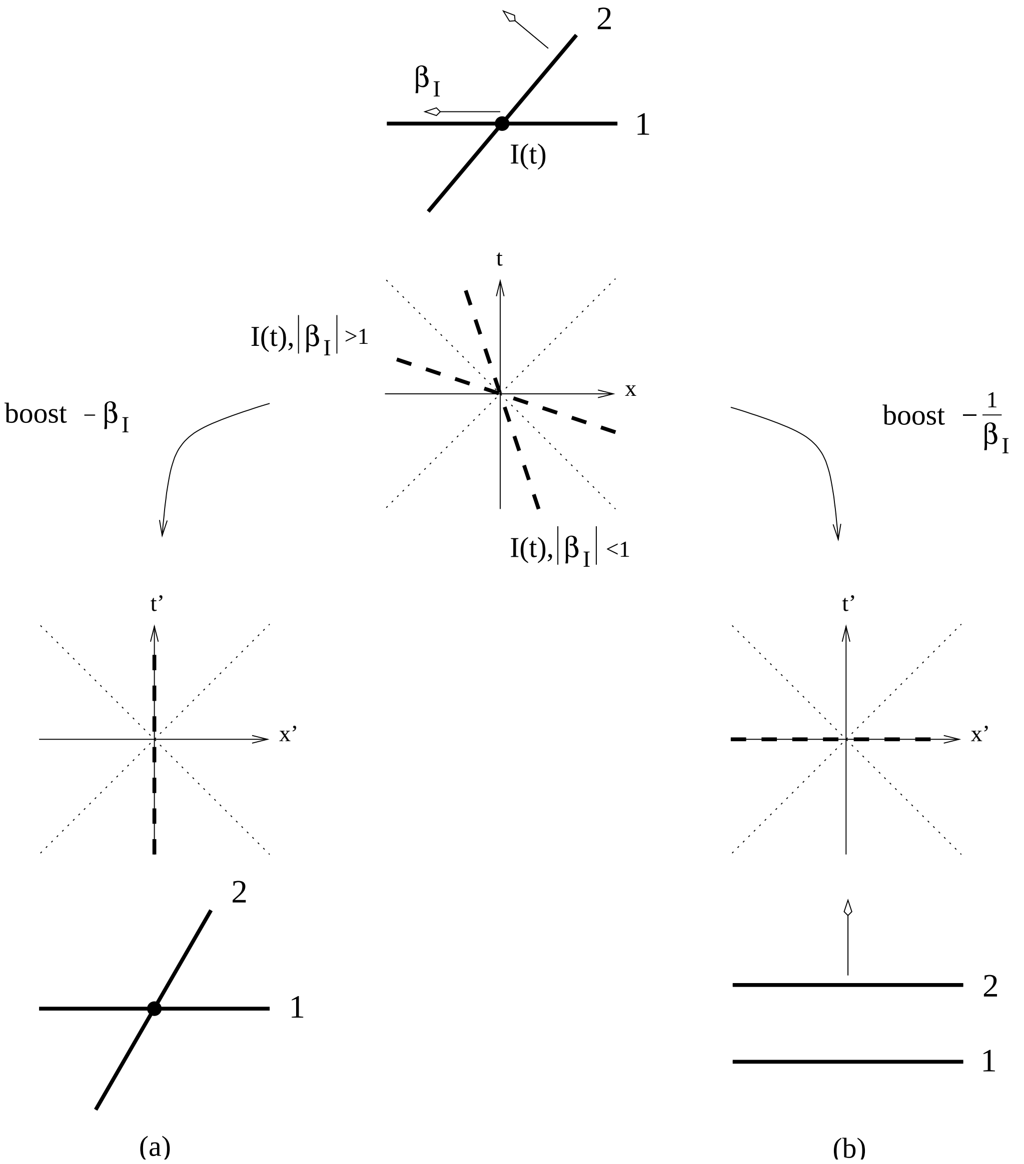

Figure 4: Boosting to the static (a) and parallel (b) configurations.

Of course, the above discussion singles out the case with $|\vec{\beta}_I| = 1$. In this case, the trajectory (3.9) lies on the lightcone and it remains null in any reference frame. We will see that considerations of supersymmetry select these configurations out as special as well.

3.2 Enter SUSY

We would like to understand the constraints imposed by demanding that the above systems involving branes in motion are supersymmetric. In section 2.2, we commented on two time-dependent, yet supersymmetric, configurations of angled branes related to the static D2-D2 brane pairs. One was constructed with the M-theory lift and the momentum arose as the geometrical quantity dual to the non-zero magnetic fields on the D2-branes, as seen in eq. (2.16). Of course, re-compactifying on some transverse direction would reduce this

M2-brane configuration to an identical system of D2-branes. One observation in eq. (2.19) was that, in this supersymmetric system of branes in motion, the intersection point moved at the speed of light. We will see below that this condition appears naturally in the supersymmetry analysis. Our results rely on the analysis of ref. [10], which discusses null intersections of M5-branes. Interesting supersymmetric configurations involving branes in motion also appear in ref. [24].

It is simplest to begin by considering the supersymmetry constraints for the cases with $|\beta_I| < 1$. As described above, these configurations are easily boosted to a frame where the intersection is static. Now, the problem of supersymmetric constraints for static systems of branes intersecting at angles has received a fair amount of study [14, 15, 16]. While the details are unimportant, the general result is that supersymmetry requires the branes to be related by a higher dimensional rotation, i.e., one involving rotations in more than one orthogonal plane. An example would be an SU(2) rotation acting in an $\mathbb{R}^4$ subspace of the ten-dimensional Minkowski background. Certainly, boosting such general configurations will produce configurations describing a system where the intersection point or surface (if the branes have common spatial dimensions) moves with a subluminal velocity. For the simple configurations considered in section 3.1, the rotation relating the branes is confined to a single plane and so it would seem that these configurations are necessarily non-supersymmetric. As a caveat, one should note that the preceding discussion applies to branes carrying no fluxes. As described in section 2.2, even for a rotation in a single plane, one can produce a supersymmetric intersection by introducing an appropriate electric flux on the branes — recall eq. (2.21). Thus, in general, our class of configurations with $|\beta_I| < 1$ admits supersymmetric solutions if one is willing to extend the discussion to intersecting Dp-F1 bound states. The supersymmetric configurations would be boosted versions of the static configurations discussed at the end of section 2.2.

For the case with $|\beta_I| > 1$, one can again boost to a canonical frame as described above, but now this frame corresponds to one where the branes are parallel but are moving relative to one another in the transverse space — see figure 4. This transverse motion breaks all of the supersymmetries. This simple observation is supported by the detailed analysis of ref. [10].

Fortunately, for the analysis of supersymmetry in the case $|\beta_I| = 1$, we can borrow from the discussion of M5-branes given in ref. [10]. As usual the supersymmetry constraints are determined from considering the compatibility of the supersymmetry projections for the individual branes, $\Gamma_i \epsilon = \epsilon$, analogous to the discussion in section 2.1. As for the general case of intersecting branes [14], the projections may be written as

$$\Gamma_2 = S\,\Gamma_1\,S^{-1}\,, \tag{3.10}$$

where S is the Lorentz transformation that relates the first brane to the second. The essential observation [10] is that for the case where $|\beta_I| = 1$ the necessary transformation can be characterized as a 'null rotation' — see, e.g., ref. [25]. For example, to produce the configuration illustrated in figure 3a, the relevant transformation is

$$S(\beta) = \exp\left[\gamma\beta\left(\Sigma_{ty} - \Sigma_{xy}\right)\right], \tag{3.11}$$

where $\Sigma_{\mu\nu}$ denotes the generator of a boost or rotation in the (x^μ, x^ν) plane. It is a straightforward exercise to show that the relevant 'rotation' parameter is $\gamma\beta$ where β is the magnitude of the brane's velocity appearing in the figure (and as usual $\gamma = 1/\sqrt{1-\beta^2}$). The defining characteristic of a null rotation is that is leaves a null vector invariant, i.e., $(1, -1, 0, \ldots)$ in the case of eq. (3.11). Hence, beginning with coincident branes, after applying a null rotation to one of them, this null vector remains common to the tangent space of both worldvolumes. That is, the intersection point or surface moves along this null direction.

Following the analysis of ref. [10], one uses the property that

$$\Gamma_1 \, S^{-1} = S \, \Gamma_1 \tag{3.12}$$

to show that compatibility of the two supersymmetry projections requires $S^2\epsilon = \epsilon$. Further, for the spinor representation, the Lorentz generators are simply $\Sigma_{\mu\nu} = \frac{1}{2}\Gamma_{\mu\nu}$, from which one finds that the square (and higher powers) of the generator appearing in the null rotation vanishes. Hence, the previous compatibility condition reduces to a simple projection along the invariant null direction. That is, for eq. (3.11), one has

$$(\Gamma_t - \Gamma_x)\,\epsilon = 0\,. \tag{3.13}$$

Such a null projection is familiar, e.g., for a gravity wave [26] or massless particle propagating along the x axis. It follows from the construction that this projection operator commutes with the original projection operator Γ_1. Hence these two projections are compatible and the brane system with a null intersection is one-quarter BPS.

Of course, this general result may have been anticipated because the null intersections arose in section 2.2 as a dual description of a one-quarter BPS configuration of parallel branes and antibranes. For example, the M-theory lift of the D2-D2 pair described by eq. (2.17) gave a pair of M2-branes with a null intersection. As a check, it is interesting to verify that the same supersymmetry conditions arise in the original configuration. For specificity consider the choice of fields $E_2 = B_2 = -\varepsilon$ and $E_1 = B_1 = 0$. Now, applying the preceding discussion to the corresponding lift to a pair of M2-branes with a null intersection, one finds:[4] $\Gamma_{txu}\epsilon = \epsilon$ and $(\Gamma_t - \Gamma_x)\epsilon = 0$. Working directly with the D2-branes, one finds that the worldvolume fluxes on the second brane are consistent with the choice $E_1 = E_1^-$ in eq. (2.10). Then the compatibility condition in eq. (2.11) becomes $\Gamma_u\epsilon = -\epsilon$ while the inequality is automatically satisfied. The original supersymmetry constraint associated with the D2-brane with vanishing fluxes is simply $\Gamma_{txu}\epsilon = \epsilon$. Combining these two projections yields $\Gamma_{tx}\epsilon = -\epsilon$, from which the desired null projection readily follows.

3.3 Worldvolume picture

Next we investigate excitations of these systems involving branes in motion. The approach followed in this section is to study the worldvolume theory. For simplicity, we consider a symmetric configuration of D-strings as illustrated in figure 5. To leading order, the

[4] Here u specifies a common worldvolume direction orthogonal to the hypersurface in which the null rotation acts, analogous to the role of y in section 2.

low energy worldvolume theory on the D-strings is the reduction of ten-dimensional U(2) super-Yang-Mills theory to two dimensions. The bosonic lagrangian, which is sufficient for our purposes, is

$$\mathcal{L} = \frac{2\pi\ell_s^2}{g}\mathrm{Tr}\,\left(-\frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}D_a\Phi^i D^a\Phi^i + \frac{1}{4}[\Phi^i,\Phi^j][\Phi^i,\Phi^j]\right). \tag{3.14}$$

Here, F_{ab} is the non-abelian U(2) field strength and Φ^i, with $i = 2,\ldots,9$, are the eight adjoint scalars. Our conventions are such that $\Phi^i = (\Phi^i)^{(n)}\tau_n$, where the hermitean generators of U(2) are chosen to be τ_0, the two-by-two identity matrix, and $\tau_{1,2,3}$, the standard Pauli matrices. For simplicity, we work in static gauge where the timelike (spacelike) coordinate on the worldvolume matches the background coordinate t (x).

The configuration shown in figure 5 is studied by introducing a background vacuum expectation value (vev) to one of the scalars, Φ^2, say, which represents displacements of the worldvolume in the y direction. The profile of the two strings in this symmetric configuration is $y = \pm(x\sin\theta + t\beta)/\cos\theta$, up to a constant shift in t or x. This then corresponds to the background scalar vev

$$\Phi_0^2 \equiv V\tau_3 = \frac{x\sin\theta + t\beta}{2\pi\ell_s^2\cos\theta}\tau_3\,. \tag{3.15}$$

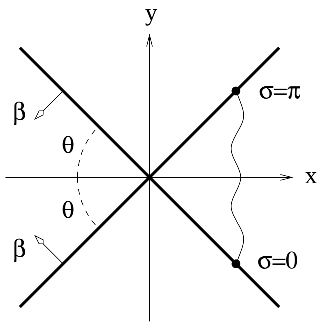

Figure 5: A symmetric D-brane configuration, with attached string.

Note that this expectation value (and all other fields zero) is a solution of the equations of motion following from eq. (3.14). One could also displace the strings along another orthogonal direction by adding an additional constant vev, e.g., $\Phi_0^3 = d\,\tau_3$, so that the D-strings do not actually intersect. This additional complication does not qualitatively modify the results below. Also note that in this symmetric configuration, the intersection between the D-strings lies on the x axis and so its apparent velocity is still given by eq. (3.8). Now, the low energy theory (3.14) will give an adequate description of the physics when the fields are slowly varying. However, we should comment that even though the backgrounds of interest may describe intersections with an apparent velocity at or near the speed of light, this does not contradict the requirement of slowly varying fields. The latter will all ways be satisfied if β and $\sin\theta$ are sufficiently small, but the latter does not restrict their ratio, which determines β_I.

With this background, gauge symmetry is spontaneously broken to U(1) $\times$ U(1). The unbroken U(1)'s are associated with the $\tau_{0,3}$ components and the corresponding fields correspond to string modes which move along one string or the other. We are most interested in the $\tau_{1,2}$ components of the fields as they correspond to strings connecting one D-string to the other. The simplest case[5] to consider is that of the transverse scalars orthogonal to the plane of the motion, i.e., Φ^i with $i = 3,\ldots,9$. We denote the corresponding off-diagonal components as the complex scalars ϕ^i. When we expand around the background (3.15),

[5]The scalar Φ^2 mixes with the gauge field, but shows qualitatively similar behavior to that found below.

the linearized equation of motion of each field is simply

$$-\Box\phi + 2V^2\phi = 0\,, \tag{3.16}$$

with what appears to be a spacetime-dependent mass term. Now, the precise behavior of the solutions depends on whether the linear combination of x and t entering V^2 is spacelike, timelike or null — which corresponds to an apparent velocity β_I which is subluminal, superluminal or null. Hence we consider each of these cases in turn:

i) Subluminal ($\beta < \sin\theta$). Here the first step is to make the coordinate transformation

$$u = t\sin\theta + x\beta\,, \qquad w = t\beta + x\sin\theta\,. \tag{3.17}$$

Note that under this transformation, the line element on the D-string worldvolume becomes

$$ds^2 = -dt^2 + dx^2 = \frac{1}{\sin^2\theta - \beta^2}\left(-du^2 + dw^2\right), \tag{3.18}$$

and so we may think of u (w) as a time (space) coordinate. In these coordinates, the equation of motion (3.16) becomes

$$\left(\partial_u^2 - \partial_w^2\right)\phi + U(w)\phi = 0\,, \tag{3.19}$$

where

$$U(w) = \frac{1}{2\pi^2\ell_s^4(\sin^2\theta - \beta^2)\cos^2\theta}\,w^2\,. \tag{3.20}$$

Making a standard separation of variables, $\phi = e^{-i\Omega u}\psi(w)$, the equation of interest reduces to

$$-\partial_w^2\psi + U(w)\psi = \Omega^2\psi\,. \tag{3.21}$$

Now eq. (3.21) has the form of the time-independent Schrödinger equation for a simple harmonic oscillator potential. Comparing to eq. (3.20), the oscillator frequency and the energy eigenvalues are:

$$\omega = \frac{1}{\pi\ell_s^2\cos\theta\sqrt{2(\sin^2\theta - \beta^2)}}\,, \qquad E = 2\Omega^2 = \left(n + \frac{1}{2}\right)\omega\,; \qquad n = 0, 1, 2, \ldots\,. \tag{3.22}$$

Further we can write out the precise solutions (see, e.g., ref. [27]) but it suffices to say that they are localized around the origin $w = 0$.

Now we observe that the transformation (3.17) essentially amounts to the Lorentz transformation that brings the branes to a static configuration, as described at the end of section 3.1. In the original frame $w = 0$ corresponds to the location of the intersection point between the two D-strings. The localization found above indicates that for the 'massless' string modes connecting the two D-strings, their excitations are carried along by the motion of the intersection in this the subluminal case. This is analogous to the results found in ref. [14] for the case of a static brane intersection. Note that for lowest-lying modes, the modes are very well confined to the vicinity of the intersection and so the higher order (cubic and quartic) interactions in the lagrangian (3.14) should be negligible. Hence the linearized equations of motion (3.16) studied here should give an accurate description of the physics.

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ii) Superluminal ($\beta > \sin\theta$). In this case, the analysis is essentially the same as above. However from eq. (3.18) we see that the role of the new coordinates is reversed, i.e., u is space coordinate while w is the time coordinate. Further since $(\sin^2\theta - \beta^2)$ has changed sign in this regime, the potential (3.20) in the effective Schrödinger equation (3.21) is now an *inverted* harmonic oscillator. Exact analytic solutions can be still written down in terms of parabolic cylinder functions (see, e.g., ref. [28]) and in accord with one's intuition, the wavefunctions are not localized near the origin.

The physical interpretation of these results is as follows. In this case, the coordinate transformation (3.17) is essentially the boost which puts the D-strings in the canonical frame where they are parallel but move relative to one another along the y axis. The failure of the wavefunctions $\psi(w)$ to be localized reflects the fact that these excitations cannot keep up with the intersection point. That is, if we construct an initial wave packet that is localized near the intersection, say, $x = 0$ at $t = 0$, this excitation falls behind the intersection which moves off to the left along $x = -|\beta_I|\, t$. Thus the wave packet will eventually enter a regime where the expectation values of the background scalars are large and so the low energy analysis becomes unreliable as the relevant mass scales are string scale.

As an aside, it is interesting to remark that if one creates a wavepacket with both $\tau_{1,2}$ components in more than one of the transverse scalars, then this excitation would generate a nontrivial coupling to the RR four-form potential [22, 29]. Hence such an excitation would introduce a nontrivial D3-brane component, which presumably gets stretched out between the D-strings.

iii) Null ($\beta = \sin\theta$). In this case, the coordinate transformation (3.17) becomes degenerate and so we must modify the analysis somewhat. It is useful to define light-cone coordinates

$$x^{\pm} = \frac{t \pm x}{\sqrt{2}}\,, \tag{3.23}$$

in terms of which the equation of motion (3.16) becomes

$$\partial_- \partial_+ \phi + \frac{\tan^2\theta}{2\pi^2 \ell_s^4} x^{+2} \phi = 0\,. \tag{3.24}$$

We can readily solve this equation with the usual separation of variables but it is useful to make yet another change of coordinates: $u = x^{+3}/\ell_s^2$. Note that with these new coordinates, the worldvolume metric becomes

$$ds^2 = -dt^2 + dx^2 = -dx^+ dx^- = \frac{1}{3}\left(\frac{\ell_s}{u}\right)^{2/3} \left(-du\, dx^-\right)\,, \tag{3.25}$$

while the equation of motion reduces to

$$\partial_- \partial_u \phi + \frac{\tan^2\theta}{6\pi^2 \ell_s^2} \phi = 0\,. \tag{3.26}$$

The latter takes the form of an ordinary Klein-Gordon equation for a scalar field with mass $M = \tan\theta/\sqrt{6}\pi\ell_s$. Hence we can easily write down solutions as

$$\phi = e^{ip_- x^-} e^{iM^2 u/p_-} = e^{ip_- x^-} e^{iM^2 x^{+3}/\ell_s^2 p_-}\,. \tag{3.27}$$

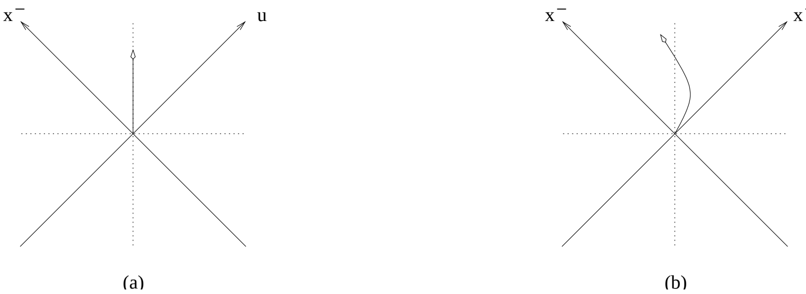

Figure 6: (a) Typical timelike geodesic in the (u, x^-) plane and (b) the corresponding trajectory in the (x^+, x^-) plane.

In these coordinates, the intersection lies at $x^+ = 0$, and the obvious interpretation of p_- is as the momentum of a plane wave along the null direction x^-. The explicit form of the wave functions (3.27) does not immediately suggest the proper physical interpretation for these results. The introduction of u is useful for these purposes. We begin by considering the equation of motion (3.26): regarding this as a massive scalar wave equation in flat Minkowski space, it is natural to construct wave packets which are peaked near timelike geodesics — see figure 6. As the flat metric $d\tilde{s}^2 = -du\, dx^-$ is related to the worldvolume metric (3.25) by a conformal transformation, the causal nature of this propagation should survive for these excitations of the D-strings. That is, that wave packets naturally follow timelike trajectories. Using the coordinate transformation $u = {x^+}^3/\ell_s^2$, we see that a typical trajectory might be $x^- = {x^+}^3/\ell_s^2$, which is easily verified to be timelike — see figure 6. Hence we see that as in the subluminal case, the excitations are 'swept' along by the null intersection. However, as in the superluminal case, the wave packets can not keep up with the intersection. Hence in this case, the excitation of the fields may not grow dramatically but invariably a wavepacket enters a regime where the expectation values of the background scalars are large and so the low energy analysis again becomes unreliable.

3.4 Macroscopic strings

One can also investigate the string modes connecting the two branes in motion by examining directly the worldsheet equations of the fundamental strings. One can naturally quantize these modes and calculate the perturbative string spectrum. However, for the purposes of gaining some physical insight into the behavior of these systems, we will satisfy ourselves here with considering particular solutions describing the behaviour of macroscopic strings connecting the two branes. In this analysis we again consider the symmetric configuration of moving D-strings examined in the last section — see figure 5.

Information about the motion of the D-strings is encoded in the boundary conditions on the worldsheet fields. Those boundary conditions for the configuration under consideration are

$$\begin{aligned} \partial_\sigma[T + \beta(Y\cos\theta + X\sin\theta)]_{\sigma=0} &= 0\,, & \partial_\sigma[T - \beta(Y\cos\theta - X\sin\theta)]_{\sigma=\pi} &= 0\,, \\ \partial_\sigma[X\cos\theta - Y\sin\theta]_{\sigma=0} &= 0\,, & \partial_\sigma[X\cos\theta + Y\sin\theta]_{\sigma=\pi} &= 0\,, \\ [T\beta + (Y\cos\theta + X\sin\theta)]_{\sigma=0} &= 0\,, & [T\beta - (Y\cos\theta - X\sin\theta)]_{\sigma=\pi} &= 0\,. \end{aligned} \qquad (3.28)$$

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The fields $\{X^i\}_{i=3\ldots9}$, that map the worldsheet into the spatial directions transverse to the plane of the configuration, have standard Dirichlet boundary conditions, and associated open string mode expansions — see, e.g., ref. [2].

When the intersection moves slower than the speed of light ($\sin\theta > \beta$), some representative solutions of $\Box X^a = 0$ may be written

$$\begin{aligned} T(\sigma,\tau) &= 2\ell_s^2 p\tau - \frac{\beta}{\cos\theta}\ell_s H(\tau)I(\sigma)\,, \qquad Y(\sigma,\tau) = \ell_s H(\tau)J(\sigma)\,, \\ X(\sigma,\tau) &= -\frac{2\beta}{\sin\theta}\ell_s^2 p\tau + \frac{\sin\theta}{\cos\theta}\ell_s H(\tau)I(\sigma)\,. \end{aligned} \tag{3.29}$$

The functions H, I and J and given by

$$\begin{aligned} H(\tau) &= A\cos\delta\tau + B\sin\delta\tau\,, \\ I(\sigma)) &= \sin\delta\sigma - \frac{\cos\theta}{\sqrt{\sin^2\theta-\beta^2}}\cos\delta\sigma\,, \\ J(\sigma) &= \sin\delta\sigma + \frac{\sqrt{\sin^2\theta-\beta^2}}{\cos\theta}\cos\delta\sigma\,, \end{aligned} \tag{3.30}$$

where δ is given by

$$\tan\pi\delta = \frac{2\cos\theta\sqrt{\sin^2\theta-\beta^2}}{\sin^2\theta-\cos^2\theta-\beta^2}\,. \tag{3.31}$$

A and B are real constants, corresponding to the simple choice of oscillators that we have excited.

In figure 7 we display a plot of the trajectory, along the x axis, of one of the endpoints of the string, calculated from eqs. (3.29). One clearly sees the oscillatory motion of the string around the moving intersection (in this direction, the two endpoints move in phase with one another). The motion of the endpoints in the y direction, perpendicular to the motion of the intersection, is also oscillatory, but now the two endpoints are in anti-phase, indicating that they pass over one another when they cross the intersection. Of course, since $\beta < \sin\theta$, this configuration could be boosted to a standstill, in which case the behaviour just described is exactly what one would expect — the confinement of modes to the region around the intersection. This localisation matches the behavior found for the microscopic strings in the U(2) gauge theory in section 3.3.

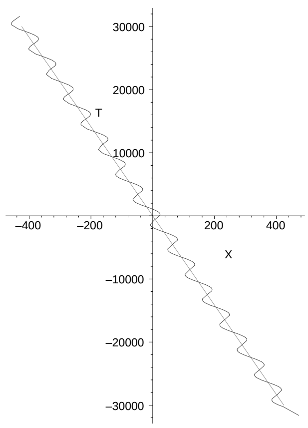

Figure 7: Oscillatory motion of the string about the moving intersection, for $\beta < \sin\theta$.

Note that in the general solution for the worldsheet modes, one finds modes with non-integer oscillator frequencies. The expression δ in eq. (3.31) is the amount by which the frequencies are shifted from integer. Notice that δ vanishes in the limit $\beta \to \sin\theta$, signalling the re-appearance of integer moded oscillators in the supersymmetric case. However, it appears that, in the same limit, the X and T mode expansions become singular. This behaviour was noticed in the mode expansions of strings joining the D2-$\overline{\text{D2}}$ pair of ref. [7], where it is an effect of the worldvolume gauge fields. There it was argued that the worldsheet CFT actually remains non-singular, so that the spacetime supersymmetric theory is well-defined.

If we continue to the regime where the intersection velocity is superluminal ($\sin\theta < \beta$), the phase shift δ becomes imaginary. The analogous solutions to those in eq. (3.29) are

$$\begin{aligned} T(\sigma,\tau) &= 2\ell_s^2 p\tau - \frac{\beta}{\cos\theta}\ell_s \bar{H}(\tau)\bar{I}(\sigma)\,, \qquad Y(\sigma,\tau) = -\ell_s \bar{H}(\tau)\bar{J}(\sigma)\,, \\ X(\sigma,\tau) &= -\frac{2\beta}{\sin\theta}\ell_s^2 p\tau - \frac{\sin\theta}{\cos\theta}\ell_s \bar{H}(\tau)\bar{I}(\sigma)\,, \end{aligned} \tag{3.32}$$

where, now,

$$\begin{aligned} \bar{H}(\tau) &= \bar{A}\cosh\bar{\delta}\tau + \bar{B}\sinh\bar{\delta}\tau\,, \\ \bar{I}(\sigma) &= \sinh\bar{\delta}\sigma + \frac{\cos\theta}{\sqrt{\beta^2-\sin^2\theta}}\cosh\bar{\delta}\sigma\,, \\ \bar{J}(\sigma) &= \sinh\bar{\delta}\sigma + \frac{\sqrt{\beta^2-\sin^2\theta}}{\cos\theta}\cosh\bar{\delta}\sigma\,, \end{aligned} \tag{3.33}$$

and

$$\tanh\bar{\delta}\pi = \frac{2\cos\theta\sqrt{\beta^2-\sin^2\theta}}{\sin^2\theta-\cos^2\theta-\beta^2}\,. \tag{3.34}$$

As we have seen, in this case it is not possible to boost the intersection to a standstill. Rather, as we know the crossed branes are now equivalent to parallel branes moving away from one another, as in figure 4b, we expect to find strings that grow without bound in time. The expressions above indeed exhibit this behaviour, implicit in the hyperbolic functions that have replaced the sines and cosines of eqs. (3.30). Therefore at large times, these functions dominate the evolution of these strings. In particular, one finds that the horizontal velocity of the string is essentially constant with $dX/dT(\sigma) \simeq \sin\theta/\beta$. Of course, this precisely matches the behavior expected from the discussion at the end of section 3.1. Hence, the string lags behind the intersection, while being stretched apart in the y direction. The gauge theory also displayed a version of this macroscopic physics, as we saw in section 3.3.

4. Discussion

In this paper, we considered two topics in the physics of D-branes. The first was generalized supersymmetry conditions for parallel D-$\overline{\text{D}}$ pairs and the second was brane intersections in motion. The connection between these configurations was that in certain situations, they naturally arose as dual descriptions of the same system.

Supertubes are a fascinating arena in which to study D-brane physics [3]–[9, 30]. They seem to provide a counter-example to several aspects of 'popular string lore,' one example being that brane-antibrane systems are necessarily nonsupersymmetric. In the case of parallel D-$\overline{\text{D}}$ pairs, we found generalized flux configurations for two-branes which allowed the systems to be one-quarter BPS. Previous studies, which produced extensions of supertube configurations, generally found the relevant electric field took the 'critical' value given in eq. (1.1). However, one of the outcomes of our analysis was that a critical electric field is not a crucial ingredient to producing a supersymmetric system. We should note that a similar conclusion is implicit in ref. [9] which studied D3-$\overline{\rm D3}$ and D4-$\overline{\rm D4}$ pairs. It is true that here we focussed only on planar configurations while the supertube allows for more interesting geometries. However, even for a supertube, one could shift the electric field away from its critical value by boosting along the axis of the tube. This boost would create an additional component to the electric field orthogonal to the axis, corresponding to introducing fundamental strings which wind around the tube. In any event, an interesting extension of our work would be to consider in detail how the tachyonic modes in the perturbative string spectrum of D-$\overline{\rm D}$ pair are lifted as the background fluxes approach their supersymmetric values.

Lifting a supersymmetric D2-$\overline{\rm D2}$ pair to eleven-dimensional M-theory produces a BPS configuration of M2-branes at an angle in which the intersection line moves at lightspeed. In considering general configurations of intersecting D-branes in motion, one finds that supersymmetry naturally selects out cases where the intersection is null. Related M5-brane configurations were previously considered in ref. [10]. Those authors provided a beautiful construction which can be applied quite generally to construct branes with null intersections by the application of a null rotation. The compatibility of the supersymmetry of the branes involves a null projection (3.13) along the tangent to the trajectory of the intersection. Similar null projections appear for related systems of branes such as superhelices [17, 18], M-ribbons [31] or giant gravitons [32].

Further analysis of excitations of the intersecting D-branes in motion gave interesting results. In the case where the intersection point moves at less than the speed of light, the excitations involving strings connecting the two branes are simply swept along by the motion of the intersection. This result appeared both in the investigations of the low energy worldvolume theory and of macroscopic strings. In the case of superluminal motion of the intersection, both approaches revealed instabilities. The macroscopic strings were unable to keep up with the intersection point and so were continually stretched as they fell behind — a result that is not surprising given the description of these intersections in a canonical frame with parallel branes moving apart. Similarly wave packets in the low energy theory would fall behind the superluminal intersection point. Analysis of the null intersections revealed a similar falling behind and stretching instability. We note that the stretching instability of the macroscopic strings need not be considered as pathological as, say, the runaway behavior of a tachyonic mode. Similar stretching instabilities have recently been discussed for closed strings in plane wave backgrounds [33]. The amplitude of the low energy excitations do not seem to show a tachyonic or runaway instability in that their amplitude does not diverge as they evolve in time. Rather, within the low energy analysis, the relevant solutions of eq. (3.21) have the asymptotic behavior $|\psi|^2 \sim 1/w$ in general.

Recently there has been a great deal of interest in time-dependent backgrounds in string theory [34, 35, 36]. One could study the systems studied above as simple time-dependent backgrounds for open strings. In many respects, the null intersections are closely related to null orbifolds [35]. In particular, the construction of both is based on a null rotation.

Although the null orbifolds are supersymmetric, one finds that in many cases these backgrounds are 'fragile' in that probes generically produce regions of strong curvature [36]. This behavior might be regarded as analogous to the stretching instability of string modes as they fall behind the intersection point. Of course for the null orbifolds, the 'probe cataclysms' can be suppressed by including a spatial translation in the orbifold group [36]. In the present case, similarly the excitation of the interbrane strings could be suppressed by increasing the threshold with an additional separation of the D-branes in the transverse space. Note, however, that the gravitational effects in the orbifold case are created independent of the value of the string coupling constant. In the present case, it seems that these string modes would only be created perturbatively when an intersection collides with modes travelling down one of the individual branes. Hence their appearance can be suppressed by reducing the string coupling. In any event, these null intersections are worthy of further study for the potential insight that they may give into the problem of time-dependent backgrounds.

Acknowledgments

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A. T-duality transformations

In general, the effect of T-duality parallel to a flat D2-D0-F1 bound state is to produce a D1-F1 bound state moving, at an angle to the T-duality direction, in the plane of the original brane system. The presence of the lower-dimensional constituents of each bound state induces a BI field strength on each D-brane. The momentum and the angle are understood to arise from the reinterpretation of certain components of the D2-brane gauge potential as new scalars transverse to the D1-brane's worldvolume. If we T-dualise in a direction x^p along the D2-brane this relationship can be written, in terms of the field strength, as — see, e.g., ref. [29]:

$$F_{ap} \longrightarrow \partial_a \Phi^p \,, \tag{A.1}$$

where the transverse scalar is related to the transverse coordinate by $\Phi^p = x^p/(2\pi\ell_s^2)$, and determines the profile of the D-string. This is sufficient to calculate the angle and momentum produced by T-duality, but is not the whole story, as it tells us nothing about the electric flux on the D-string. The simplest way to calculate this is to infer the result from the action of the T-duality on the worldsheet boundary conditions for the open strings ending on the D-string or the D2-brane. The details of this procedure can be found in, e.g., ref. [2, 23], and give values for the angle and momentum that agree with those determined from eq. (A.1). We start with the D-string configured as in figure 8, with an electric field denoted by e. Then the T-dual configuration is a D2-brane filling the (x, y) plane, with BI field strength

$$F = \tilde{E}\mathrm{d}x \wedge \mathrm{d}t + E\mathrm{d}y \wedge \mathrm{d}t + B\mathrm{d}y \wedge \mathrm{d}x\,, \tag{A.2}$$

where, if the T-duality was made along the x axis,

$$\tilde{E} = \frac{\beta}{2\pi\ell_s^2 \sin\theta}\,, \qquad E = \frac{e}{\gamma \sin\theta}\,, \qquad B = \frac{\cot\theta}{2\pi\ell_s^2}\,. \tag{A.3}$$

Alternatively, T-duality along the y axis gives

$$\tilde{E} = \frac{e}{\gamma\cos\theta}\,, \qquad E = -\frac{\beta}{2\pi\ell_s^2\cos\theta}\,, \qquad B = -\frac{\tan\theta}{2\pi\ell_s^2}\,. \tag{A.4}$$

Note that $\gamma = 1/\sqrt{1-\beta^2}$, as usual.

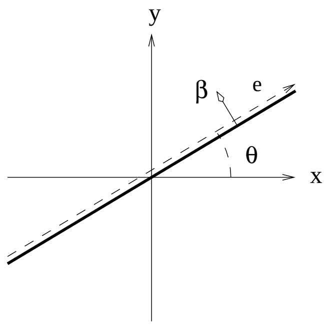

Figure 8: The dual D-string with flux.

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